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II. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Let ACB be the triangle; choose $BC=a$ for the axis of abscissas, and CA for that of ordinates. Any circumscribed ellipse is of the form $y^2 + Axy + Bx^2 + Cy + Dx = 0$; and since $(a, 0)$ and $(0, b)$ are points of the ellipse, we have $C = -b$, $D = -Ba$, and the above equation reduces to $y^2 + Axy + Bx^2 - by - Bax = 0$.

Transforming it to the center of the ellipse as origin, it reduces to

$$y^2 + Axy + Bx^2 - \frac{Bb^2 - ABab + B^2a}{4B - A^2} = 0.$$

The area is $= \pi \sin C$. $\frac{Bb^2 - ABab + B^2a}{4B - A^2} = m$. Developing $\frac{\partial m}{\partial A} = 0$, and $\frac{\partial m}{\partial B} = 0$, we find $A = \frac{b}{a}$ and $B = \frac{b^2}{a^2}$, and thus find the ellipse of minimum area to be, $a^2y^2 + abxy + b^2x^2 - a^2by - ab^2x = 0$. The center is the point $(a/3, b/3)$. The maximum ellipse about the triangle is concentric, and its equation is

$$a^2y^2 + abxy + b^2x^2 - a^2by - ab^2x + \frac{a^2b^2}{4} = 0.$$

Also solved by C. N. Schmall, and V. M. Spunar.

280. Proposed by C. N. SCHMALL, 89 Columbia Street, New York.

Find the envelope of the system of spheres

$$\left. \begin{aligned} (x-a)^2 + (y-b)^2 + z^2 &= r^2 \\ a^2 + b^2 &= c^2 \end{aligned} \right\}.$$

Solution by J. SCHEFFER, A. M., Hagerstown, Md.; G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.; V. M. SPUNAR, Pittsburg, Pa., and the PROPOSER.

Differentiating $(x-a)^2 + (y-b)^2 + z^2 = r^2$ and $a^2 + b^2 = c^2$ with reference to a as the independent variable, we have $(x-a) + (y-b)\frac{\partial b}{\partial a} = 0$, and $a + b\frac{\partial b}{\partial a} = 0$; from the second equation we get $\frac{\partial b}{\partial a} = -\frac{a}{b}$, and substituting in the first we get $(x-a) - \frac{a}{b}(y-b) = 0$, whence $b = \frac{ay}{x}$ and combining this with $a^2 + b^2 = c^2$, we get $a^2 = \frac{c^2x^2}{x^2 + y^2}$, $b^2 = \frac{c^2y^2}{x^2 + y^2}$, and substituting this in the first given equation, we have, after some easy reductions:

$$x^2 + y^2 + z^2 - 2c\sqrt{(x^2 + y^2)} = r^2 - c^2.$$

This is, as can easily be seen, the equation of the surface of a ring, the central line of which is a circumference whose radius is $= c$, and the perpendicular section of a circle whose radius is $= \sqrt{(2c^2 - r^2)}$.